(In investigation 1 and 2, do your best, if I am not here I know these are going to confuse you. I have attached the answers at the end… use them only after reading the problems thoroughly and trying them with your partner) You will get more practice Tomorrow! ;)

Investigation 1

In planning the Kent County summer jobs program, county officials must consider the relationships between the pay offered for each student, the number of students who could be hired, and the number of students interested in the work opportunity. Analysis of these relationships involves work with systems of functions. As you complete this investigation, look for answers to this question:

What strategies are useful in solving problems that involve links between two functions—one a linear function and one an inverse variation function?

The problem-solving process involves two major steps. First, you have to identify independent and dependent variables and the functions that relate those variables. Then you have to use the functions to answer questions about the variables.

1 Kent County has $200,000 to spend on student salaries.

a. How many student workers can be hired if the county pays $2,000 per worker for a summer contract covering eight weeks? What if the county pays only $1,500 per worker? What if the county pays only $1,000 per worker?

b. If the pay per worker is represented by p, what function $h(p)$ shows how the number of students who could be hired depends on the level of pay offered?

c. Sketch a graph of the function $h(p)$ and write a brief description of the way $h(p)$ changes as p increases.

2 If the jobs program offers very low pay, then few students will be interested in the work opportunity. After doing a survey in one local high school, Kent County officials arrived at the following estimates of the relation between summer pay rate and number of students they can expect to apply for the jobs.

Summer Jobs Program

Pay Offered (in dollars)	500	1,000	1,500	2,000	2,500
Expected Applicants	55	100	155	210	255

a. Does the pattern in the data table seem reasonable? Why or why not?

b. What function $s(p)$ would be a good model of the relationship between the number of students who will apply for the jobs and the level of pay offered?

c. Sketch a graph of $s(p)$ and write a brief description of what it shows about the way the number of job applicants changes as pay increases.

3. The decision to be made by Kent County summer jobs officials is how much pay to offer for the eight-week summer work contracts. Both the number of students who could be hired and the number of students who would be interested in the summer work depend on the pay rate p.

   a. Write equations and inequalities that match the following questions about the jobs program, and then estimate or find exact values for solutions.

      i. For what pay rate(s) will the number of students who can be hired equal the number of students who would be interested in the work?

      ii. For what pay rate(s) will the number of students who can be hired be less than the number of students who would be interested in the work?

      iii. For what pay rate(s) will the number of students who can be hired be greater than the number of students who would be interested in the work?

   b. When the head of the Kent County summer jobs program had to report to the county council about program plans, he wanted a visual aid to help in explaining the choice of a pay rate to be offered to student workers. Sketch a graph showing how both $h(p)$ and $s(p)$ depend on p and explain how the graph illustrates your answers to the questions in Part a.

Check Your Understanding

Each year, the Wheaton Boys and Girls Club sells fresh Christmas trees in December to raise money for sports equipment. They have $2,400 to use to buy trees for their lot; so the number of trees they can buy depends on the purchase price per tree p, according to the function $n(p) = \frac{2,400}{p}$. Experience has shown that (allowing for profit on each tree sold) the number of trees that customers will purchase also depends on p with function $c(p) = 300 - 6p$.

a. Write equations and inequalities that match the following questions about prospects of the tree sale and then estimate solutions.

   i. For what price per tree will the number of trees that can be bought equal the number of trees that will be sold?

   ii. For what price per tree will the number of trees that can be bought be greater than the number of trees that will be sold?

   iii. For what price per tree will the number of trees that can be bought be less than the number of trees that will be sold?

b. Sketch graphs showing how the supply and demand functions $n(p)$ and $c(p)$ depend on price per tree and explain how the graphs illustrate your answers to the questions of Part a.

Investigation 2

In your work on the problems of Investigation 1, you developed strategies for solving equations involving linear and inverse variation functions. As you work on the problems in this investigation, look for answers to this question:

What strategies are effective in solving equations that relate linear and quadratic functions?

In most businesses, one of the most important tasks is setting prices for the goods or services that are being offered for sale. For example, consider the case of producers who have a contract to bring a musical production to a summer theater.

They have to estimate costs of putting on the show, income from ticket sales and concessions, and the profit that can be made. Values of these variables depend on the number of tickets sold and the prices charged for tickets.

1. Data from a market survey suggest the following relationship between ticket price and number of tickets sold.

Relationship between Ticket Price and Ticket Sales

Price (in dollars)	5	10	15	20	30	40
Tickets Sold	2,300	2,000	1,700	1,500	1,050	500

a. After plotting the data and experimenting to find a function model for the pattern, the business planners proposed the function $s(p) = 2{,}500 - 50p$ for this demand pattern.

i. What do p and $s(p)$ represent in that function?

ii. Is that function reasonable? Can you produce a better model?

b. What do the numbers 2,500 and −50 tell about the way ticket sales depend on ticket price?

2. Based on the relationship between ticket price and number of tickets sold, the show planners figured that income could be predicted from ticket price, as well. They reasoned that since income is equal to the product of price per ticket and number of tickets sold, $I(p) = p(2{,}500 - 50p)$.

a. Test this function rule by calculating the predicted income from ticket sales in two ways.

i. First, use the data in Problem 1 to estimate income if the ticket price is set at \$10, \$20, and \$40.

ii. Then use the function to calculate predicted ticket income for the same ticket prices.

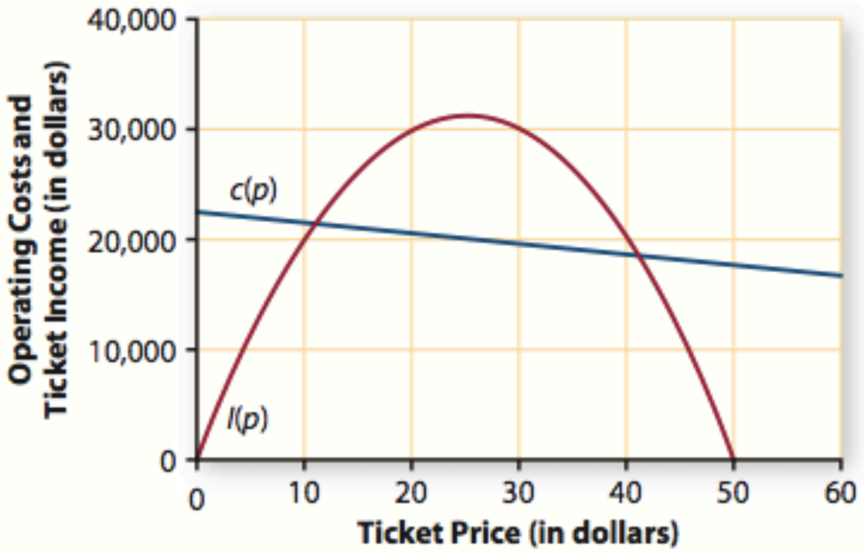
3 The next step in making business plans for the production was to estimate operating costs. Some costs were fixed (for example, pay for the cast and rent of the theater), but other costs would depend on the number of tickets sold s (for example, number of ushers and ticket takers needed). After estimating all of the possible operating costs, the function $c(s) = 17{,}500 + 2s$ was proposed.

a. According to that rule, what are the fixed operating costs and the costs per customer?

b. To show how operating costs depend on ticket price, Daniel proposed the function $c(p) = 17{,}500 + 2(2{,}500 - 50p)$. Is this rule correct? Why or why not?

c. Minta suggested that the expression $17{,}500 + 2(2{,}500 - 50p)$ in Part b could be simplified to $22{,}500 - 100p$. Is that correct? Why or why not?

4 The crucial step in business planning came next—finding out the way that ticket price would affect profit. The following graph shows how income and operating cost depend on ticket price and how they are related to each other.



a. Use the graph to estimate answers for the following questions, and explain how you arrive at each estimate.

i. For what ticket price(s) will operating cost exceed income?

ii. For what ticket price(s) will income exceed operating cost?

iii. For what ticket price(s) will income equal operating cost?

b. Use expressions in the income and operating cost functions to write and solve an equation that helps in locating the *break-even* point(s)—the ticket prices for which income exactly equals operating cost.

5 It is likely that the show producers want to do more than break even. They will probably seek maximum profit.

a. Use the income and operating cost functions to write a function showing how profit depends on ticket price. Write the function in two equivalent forms—one that shows the expressions for income and cost and another that is simplest for calculation of profit.

b. Use the profit function to estimate the maximum profit plan—the ticket price that will lead to maximum profit and the dollar profit that will be made at that price.

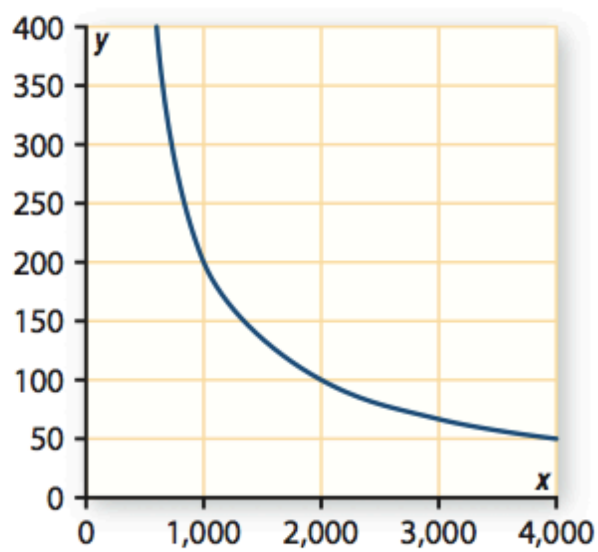
c. Use the results from Part b to calculate the number of tickets sold and the operating cost in the maximum profit situation.

ANSWERS Investigation 1

1 **a.** 100 workers can be hired at $2,000 per worker, 133 at $1,500 per worker, and 200 at $1,000 per worker.

b. $h(p) = \dfrac{200{,}000}{p}$

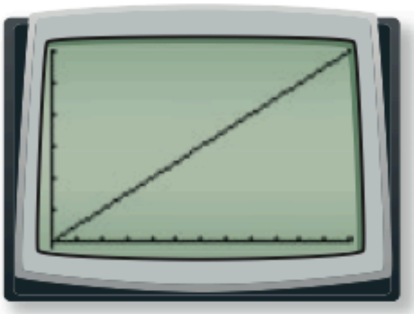
c. As p increases at a constant rate, the number of potential workers decreases at a decreasing rate.

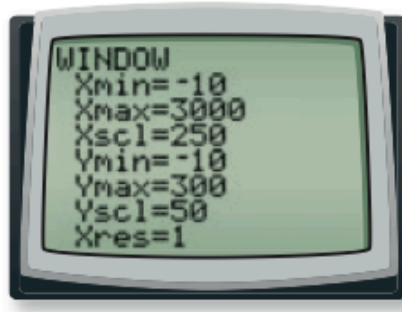


2 **a.** Yes, the more money that is offered, the more applicants one would expect. Some students might suggest that the relationship increases at a larger constant rate or an increasing rate.

b. $s(p) = 0.1p$ models the relationship well.

c. The graph shows that for every increase of one in p, $s(p)$ will increase by 0.1. Perhaps a more logical representation in this case is that for every increase of \$10 in the pay offered, Kent County should expect one more applicant.





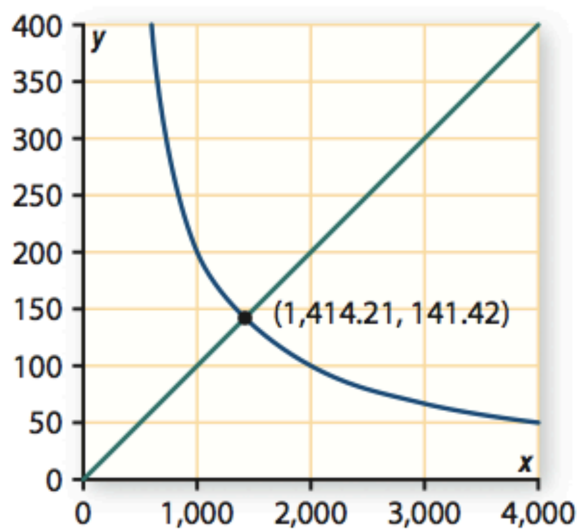
3 **a.** **i.** $\dfrac{200{,}000}{p} = 0.1p$; $p = \$1{,}414.21$

ii. $\dfrac{200{,}000}{p} < 0.1p$; $p > \$1{,}414.21$

iii. $\dfrac{200{,}000}{p} > 0.1p$; $p < \$1{,}414.21$

b. $h(p) = \dfrac{200{,}000}{p}$ and $s(p) = 0.1p$

The intersection point represents the case of the pay rate when the number of students who can be hired equals the number interested in work. The curve representing $y = \dfrac{200{,}000}{x}$, the number of students who can be hired, is above the line representing the number of students interested when $x \leq 1{,}414$ and below the line when $x \geq 1{,}415$ students.



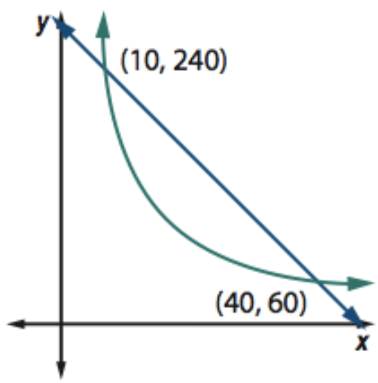
Check Your Understanding

a. **i.** $\frac{2{,}400}{p} = 300 - 6p$; $p = 10$ or $p = 40$

ii. $\frac{2{,}400}{p} > 300 - 6p$; $p < 10$ or $p > 40$

iii. $\frac{2{,}400}{p} < 300 - 6p$; $10 < p < 40$

b. The graph below shows the two graphs, the linear, number of customers (or demand), function and the inverse variation, number of trees available (or supply), function. Their intersection points indicate the prices where supply equals demand. When the linear graph is "above" the inverse variation graph, demand exceeds supply, and when the inverse variation graph is "above" the linear graph, supply exceeds demand.



ANSWERS Investigation 2

1 **a.** This fits reasonably well: p represents the ticket price, $s(p)$ represents the projected number of tickets sold. Using 2,508 instead of 2,500 is actually a slightly better fit with the given data.

b. The 2,500 may indicate the capacity of the theatre or that if the tickets were given away for free that 2,500 people would take tickets.
The −50 indicates that according to the function, for every $1 increase in ticket price, 50 fewer people will buy tickets for the show.

2 **a.** **i.** (*Price, Income*): (10, 20,000), (20, 30,000), and (40, 20,000)

ii. (*Price, Income*): (10, 20,000), (20, 30,000), and (40, 20,000)

b. Yes, the distributive property was used, which makes the two expressions equivalent. (Students may also compare graphs and tables of both functions to explore the equivalence.)

3 **a.** The fixed costs are \$17,500. The cost per customer is \$2.

b. Yes, because the $2{,}500 - 50p$ expression can be substituted for s in the first proposed cost function.

c. Yes, the distributive property can be used followed by a combining of like terms.

4 **a.** **i.** Operating costs will exceed income when $p < 11$ or $p > 41$.

ii. Income will exceed operating cost when $11 < p < 41$.

iii. When $p \approx 41$ or $p \approx 11$, income will equal operating cost.

b. $22{,}500 - 100p = 2{,}500p - 50p^2$

$50p^2 - 2{,}600p + 22{,}500 = 0$

$50(p^2 - 52p + 450) = 0$

$p = \frac{52}{2} \pm \frac{\sqrt{904}}{2}$

$p \approx 26 \pm 15$

The break-even points are around \$11 and \$41.

5 It is likely that the show producers want to do more than break even. They will probably seek maximum profit.

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b. Use the profit function to estimate the maximum profit plan—the ticket price that will lead to maximum profit and the dollar profit that will be made at that price.

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