# On Your Own

## Applications

1. The tenth-grade class officers at Columbus High School want to have a special event to welcome the incoming ninth-grade students. For \$1,500, they can rent the Big Ten entertainment center for an evening. Their question is what to charge for tickets to the event so that income from ticket sales will be very close to the rental charge.

   a. Complete a table illustrating the pattern relating number of ticket sales $n$ required to meet the "break-even" goal to the price charged $p$. Then write a rule relating $n$ to $p$.

| Price $p$ (in dollars) | 1 | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|---|
| Tickets Sales Needed $n$ | 1,500 | 500 | | | | |

   b. Study entries in the following table showing the class officers' ideas about how price charged $p$ will affect number of students $s$ who will buy tickets to the event. Then write a rule relating $s$ to $p$.

| Price $p$ (in dollars) | 0 | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|---|
| Likely Ticket Sales $s$ | 600 | 540 | 480 | 420 | 360 | 300 |

   c. Write and solve an equation that will identify the ticket price(s) that will attract enough students for the event to meet its income goal. Illustrate your solution by a sketch of the graphs of the functions involved with key intersection points labeled by their coordinates.

2. When Coty was working on his Eagle Scout project, he figured he needed 60 hours of help from volunteer workers. He did some thinking to get an idea of how many workers he might need and how many volunteers he might be able to get.

   a. He began by assuming that each volunteer would work the same number of hours. In that case, what function $w(h)$ shows how the number of volunteer workers needed depends on the number of hours per worker $h$?

   b. Coty estimated that he could get 25 volunteers if each had to work only 3 hours and only 15 volunteers if each had to work 5 hours. What linear function $v(h)$ matches these assumptions about the relationship between the number of volunteers and the number of hours per worker $h$?

   c. Write and solve an equation that will help in finding the number of hours per worker and number of workers that Coty needs. Illustrate your solution by a sketch of the graphs of the functions involved with coordinate labels on key points.

3. Use symbolic reasoning to find all solutions for these equations. Illustrate each solution by a sketch of the graphs of the functions involved, labeling key points with their coordinates.

   **a.** $x + 5 = \frac{6}{x}$

   **b.** $-0.5x = \frac{4}{x}$

   **c.** $1.5x = \frac{24}{x}$

   **d.** $10 - x = \frac{7}{x}$

4. Use symbolic reasoning to find all solutions for the equation $\frac{4}{x} + 1 = 2 - x$. Illustrate the solution by a sketch of the graphs of the functions involved, labeling key points with their coordinates.

5. In making business plans for a pizza sale fund-raiser, the Band Boosters at Roosevelt High School figured out how both sales income $I(n)$ and selling expenses $E(n)$ would probably depend on number of pizzas sold $n$. They predicted that $I(n) = -0.05n^2 + 20n$ and $E(n) = 5n + 250$.

   **a.** Estimate value(s) of $n$ for which $I(n) = E(n)$ and explain what the solution(s) of that equation tell about prospects of the pizza sale fund-raiser. Illustrate your answer with a sketch of the graphs of the two functions involved, labeling key points with their coordinates.

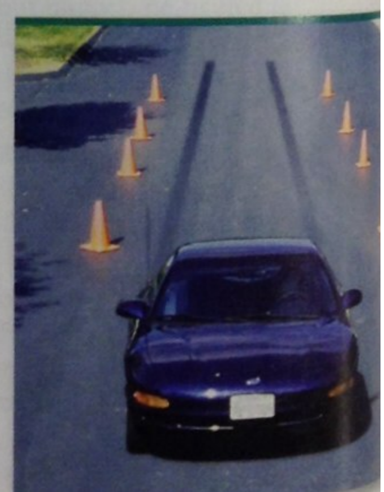
   **b.** Write a rule that gives predicted profit $P(n)$ as a function of number of pizzas sold and use that function to estimate the number of pizza sales necessary for the fund-raiser to break even. Illustrate your answer with a sketch of the graph of the profit function, labeling key points with their coordinates.

   **c.** Use the profit function to estimate the maximum profit possible from this fund-raiser. Then find number of pizzas sold, income, and expenses associated with that maximum profit situation.

6. The stopping distance $d$ in feet for a car traveling at a speed of $s$ miles per hour depends on car and road conditions. Here are two possible stopping distance formulas: $d = 3s$ and $d = 0.05s^2 + s$.

   **a.** Write and solve an equation to answer the question, "For what speed(s) do the two functions predict the same stopping distance?" Illustrate your answer with a sketch of the graphs of the two functions, labeling key point(s) with their coordinates.

   **b.** In what ways are the patterns of change in stopping distance predicted by the two functions as speed increases similar and in what ways are they different? How do the function graphs illustrate the patterns you notice?



7 Use symbolic reasoning to find all solutions for these equations. Illustrate each solution by a sketch of the graphs of the functions involved, labeling key points with their coordinates.

**a.** $2x = 2x^2 - 4x$

**b.** $2x^2 - 4x = 4 - 2x$

**c.** $x^2 - 4x - 5 = 2x + 2$

**d.** $-3 - x = x^2 + 3x + 1$

8 Give specific examples of an equation involving one linear and one quadratic function that illustrate cases a–c described below. In each case, give a sketch showing how graphs of the two functions involved in the equation are related to each other. Explain how that relationship illustrates the number of solutions to the equation.

**a.** Two distinct solutions

**b.** Exactly one solution

**c.** No solutions with real numbers

9 Find all points of intersection of graphs of the following linear functions with the circle $(x - 4)^2 + (y - 1)^2 = 10$.

**a.** $y = 2$

**b.** $y = x + 1$

**c.** $y = -x - 3$

**d.** $y = x$

10 Find all points of intersection of the graphs of the following pairs of functions.

**a.** $y = x^2$ and $y = -4x^2 + 5$

**b.** $y = x^2 + 6x$ and $y = 0.5x^2$

**c.** $y = x^2 + 3x - 4$ and $y = -x^2 + x + 6$

## Connections

11 In your early study of systems of linear equations, you found the intersection point of graphs for linear functions like $y = mx + n$ and $y = ax + b$. You found that you could solve such systems by setting $mx + n = ax + b$ and solving for $x$. You used a similar strategy in the investigations of this lesson to solve systems of equations like $y = mx + b$ and $y = \frac{k}{x}$ and like $y = mx + b$ and $y = ax^2 + bx + c$. Compare the solution possibilities for these three types of systems by answering Parts a–c.

**a.** How many solutions can there be for a system of two linear equations with two variables? Draw sketches of graphs showing the different possibilities.